THERMODYNAMICS – Exam #2

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Date: Monday, January 12th 2009; 6:00PM
Location: ENG Attic
Instructor: Dr. Wassim HABCHI
Notes: One A4 format Cheat Sheet
Value: 20% of Total Grade
Time: 90 Minutes

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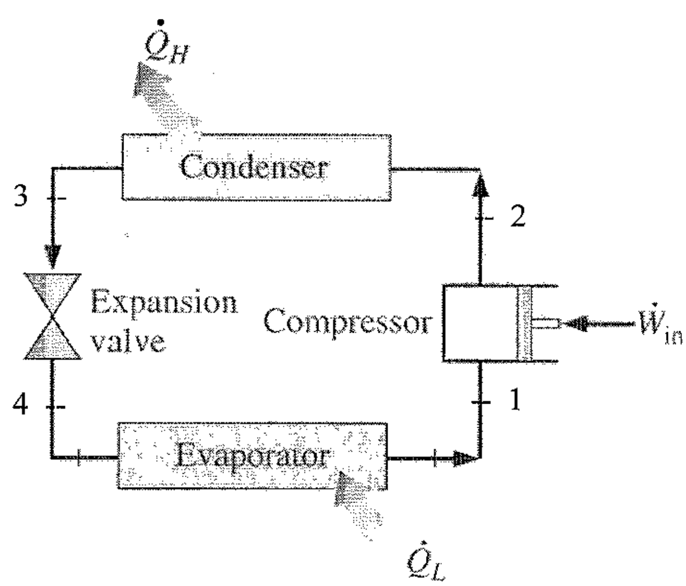
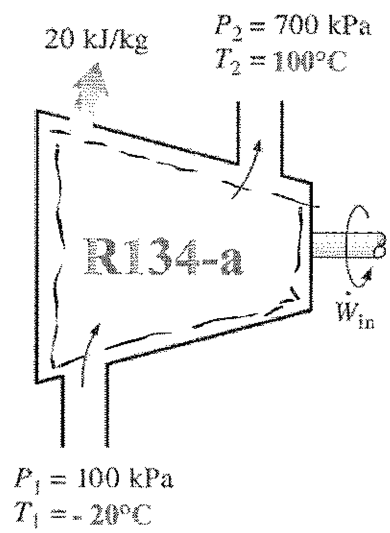
Problem I (40 points)

Household refrigerators usually operate on a closed-loop cycle as shown in the figure below. The working fluid (or refrigerant), at a superheated state, enters a compressor where it is compressed to a high pressure. The power supplied by the compressor is $\dot{W}_{in}$. The high pressure refrigerant then enters the condenser (a heat exchanger) where it condenses by rejecting an amount of heat $\dot{Q}_H$ to the surrounding atmosphere / Air in the kitchen. The refrigerant leaves the condenser as a saturated liquid. It is then throttled to a low pressure in an expansion valve before entering the evaporator (a heat exchanger) where, this time, it evaporates by extracting an amount of heat $\dot{Q}_L$ from the air **inside** the refrigerator. This is how the latter is cooled. Finally, the evaporated refrigerant reenters the compressor and the cycle repeats itself.



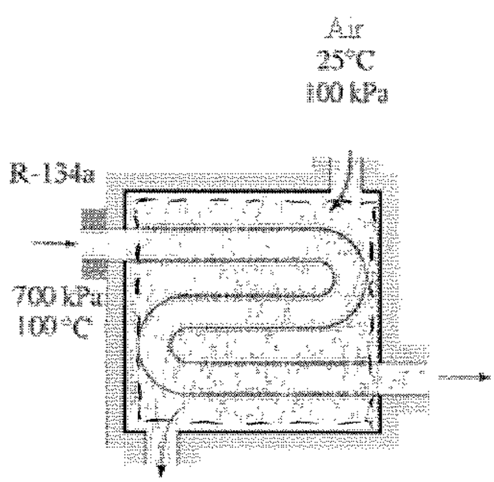
Consider a household refrigerator with refrigerant R-134a as the working fluid.

1. Compressor:



Refrigerant R-134a enters the compressor at P_1=100 KPa and T_1=-20°C. A heat loss of 20 KJ/Kg occurs during the process and the power supplied by the compressor shaft is $\dot{W}_{in} = 3.5KW$. Determine the mass flow rate of refrigerant $\dot{m}_R$ if it is to leave the compressor at P_2=700KPa and T_2=100°C.

2. Condenser:



The condenser is a heat exchanger where the refrigerant condenses by releasing an amount of heat $\dot{Q}_H$ to the surrounding Air. The refrigerant enters the condenser at (P_2,T_2) with a mass flow rate $\dot{m}_R$, and leaves it as a saturated liquid. Neglecting any pressure drops in the condenser, determine the exit temperature of the surrounding Air if its mass flow rate is 0.5 Kg/s and it enters the condenser at a temperature of 25°C and 100 KPa.

3. Expansion Valve:

After leaving the condenser, refrigerant R134-a is throttled in an expansion valve to a pressure of 100 KPa. Determine the quality of the refrigerant at the exit of the expansion valve and the temperature drop associated to this process.

4. Evaporator:

The evaporator is also a heat exchanger (similar to condenser) where the refrigerant exchanges heat with the Air inside the refrigerator. In fact, Air enters the evaporator at 100KPa and 25°C and is cooled off by loosing heat to the refrigerant. Neglecting any pressure drop in the evaporator determine the mass flow rate of air in the evaporator if the inside Air of the refrigerator is to leave the evaporator at 0°C.

5. Coefficient Of Performance:

The Coefficient Of Performance (COP) of a refrigerator is defined as:

$$COP = \frac{\text{Desired output}}{\text{Required input}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$$

Determine the COP of the above refrigerator.

Solution:

1) 1st Law of thermodynamics:

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{sys}}{dt}$$

But since it's a single stream steady-state device $\frac{dE_{sys}}{dt} = 0$ and $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$

$$\Rightarrow \dot{E}_{in} = \dot{E}_{out}$$

$$\cancel{\dot{Q}_{in}} + \dot{W}_{in} + \dot{E}_{mass,in} = \dot{Q}_{out} + \cancel{\dot{W}_{out}} + \dot{E}_{mass,out}$$

$$\dot{W}_{in} + \dot{m}_R h_1 = \dot{m}_R q_{out} + \dot{m}_R h_2$$

$$\Rightarrow \dot{m}_R = \frac{\dot{W}_{in}}{h_2 - h_1 + q_{out}}$$

@1: $h_1 = 239{,}5\ KJ/Kg$ (table A-13)
@2: $h_2 = 338{,}4\ KJ/Kg$ (" ")

$$\rightarrow \dot{m}_R = \frac{3{,}5}{338{,}4 - 239{,}5 + 20} = \boxed{0{,}0294\ Kg/s}$$

2) 1^{st} Law of thermodynamics applied to the condenser:

$$\dot m_R \cdot h_{R,in} + \dot m_A \times h_{A,in} = \dot m_R h_{R,out} + \dot m_A \cdot h_{A,out}$$

$$\dot m_A \cdot h_{A,out} = \dot m_R (h_{R,in} - h_{R,out}) + \dot m_A h_{A,in}$$

$$h_{A,out} = \frac{\dot m_R}{\dot m_A} (h_{R,in} - h_{R,out}) + h_{A,in}$$

but $h_{R,in} = 338{,}4$ KJ/Kg

$h_{R,out} = h_f @ 700\text{KPa} = 88{,}82$ KJ/Kg → Table A-12

$h_{A,in} = 298{,}18$ KJ/Kg → Table A-17

$$\Rightarrow h_{A,out} = \frac{0{,}0294}{0{,}5} (338{,}4 - 88{,}82) + 298{,}18 = 312{,}86 \simeq 313 \text{ KJ/Kg}$$

From Table A-17 → $\boxed{h_{A,out} = 312\text{K} = 39°C}$

3) At the exit of the expansion valve: P = 100KPa and h = 88,82 KJ/Kg

since $h_f < h < h_g$ → Sat. Liq. vap. mixture

$$x_e = \frac{88{,}82 - 17{,}28}{217{,}16} = \boxed{0{,}33}$$ and $T_e = T_{sat} @ 100\text{KPa} = -26{,}37°C$

$\Rightarrow \Delta T = T_e - T_i$ but $T_i = T_{sat} @ 700\text{KPa} = 26{,}69°C$

$$\Rightarrow \Delta T = -26{,}37 - 26{,}69 = \boxed{-53{,}06\ °C}$$

4) 1^{st} Law of thermodynamics applied to the evaporator:

$$\dot m_A \cdot h_{A,in} + \dot m_R h_{R,in} = \dot m_A \times h_{A,out} + \dot m_R \cdot h_{R,out}$$

$$\dot m_A (h_{A,in} - h_{A,out}) = \dot m_R (h_{R,out} - h_{R,in})$$

$$\Rightarrow \dot m_A = \dot m_R \frac{(h_{R,out} - h_{R,in})}{h_{A,in} - h_{A,out}}$$

but $h_{R,in} = 88{,}82$ KJ/Kg and $h_{R,out} = h_1 = 239{,}5$ KJ/Kg

$h_{A,in} = 298,18 \ KJ/Kg$ and $h_{A,out} = 273 \ KJ/Kg$ (A–17)

$$\Rightarrow \dot{m}_A = \frac{0,0294 \ (239,5 - 88,82)}{298,18 - 273} = \boxed{0,176 \ Kg/s}$$

5) $\dot{Q}_L$ is the amount of heat absorbed from the Air in the evaporator and it equal to:

$$\dot{Q}_L = \dot{m}_A (h_{A,in} - h_{A,out}) = 0,176 \times (298,18 - 273)$$

$$\dot{Q}_L = 4,43168 \ KW$$

And $COP = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{4,43168}{3,5} = \boxed{1,266}$

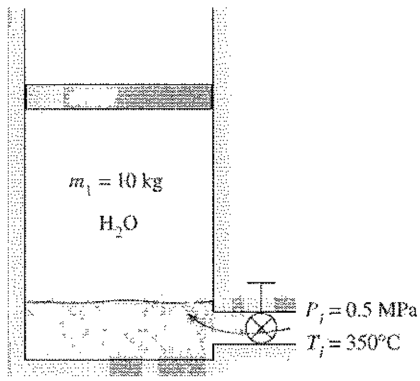
Problem II (30 points)

An insulated, vertical piston cylinder device initially contains 10 Kg of water, 6 Kg of which is in the vapor phase. The piston moves freely inside the cylinder, and it has a mass of 5 kg and a surface area of 5cm^2. Now steam at 0.5 MPa and 350°C is allowed to enter the cylinder from a supply line until all the liquid in the cylinder has vaporized. Determine:

a) The final temperature in the cylinder

b) The mass of the steam that has entered the system

Hint: Combine the moving boundary work and internal energy terms into one term!!!



Take g=10 m/s^2

Solution:

a) An equilibrium of forces on the piston gives:

$$P_1 \times A = mg + P_{atm} \times A \rightarrow P_1 = \frac{mg}{A} + P_{atm} = \frac{5\times10}{5\times10^{-4}} \times 10^{-3} + 100$$

$$\boxed{P_1 = 200 \text{ KPa}}$$

At the final state the water is in the saturated vapor state and the pressure $P_2 = P_1$ since the piston is moving freely and it has a constant mass.

$$\Rightarrow T_2 = T_{sat} @ 200\text{KPa} = \boxed{120,21°C}$$

b) This is an unsteady-flow system. The mass balance gives:

$$m_{in} - \cancel{m_{out}} = \Delta m_{sys} = m_2 - m_1 \implies \boxed{m_{in} = m_2 - m_1}$$

And the energy balance gives:

$$E_{in} - E_{out} = \Delta E_{sys}$$

$$\cancel{Q_{in}} + \cancel{W_{in}} + m_{in} h_{in} - \cancel{Q_{out}} - \cancel{W_{out}} - \cancel{m_{out} h_{out}} - W_b = \Delta U + \cancel{\Delta KE} + \cancel{\Delta PE}$$

$$m_{in} h_{in} = \Delta U + W_b = \Delta U + P\Delta V \quad \text{since } P = cte$$

$$m_{in} h_{in} = \Delta H = H_2 - H_1$$

$$m_{in} h_{in} = m_2 h_2 - m_1 h_1$$

$$m_{in} h_{in} = (m_{in} + m_1) h_2 - m_1 h_1$$

$$m_{in} (h_{in} - h_2) = m_1 (h_2 - h_1)$$

$$m_{in} = m_1 \frac{(h_2 - h_1)}{h_{in} - h_2}$$

@1: $P_1 = 200\,kPa$ and $x_1 = 0{,}6 \longrightarrow h_1 = h_f + x_1 h_{fg} = 504{,}71 + 0{,}6 \times 2201{,}6$

$\boxed{h_1 = 1825{,}67\ kJ/kg}$ (Table A-5)

@2: $P_2 = 200\,kPa$ and sat. vap $\longrightarrow h_2 = h_g @ 200\,kPa = \boxed{2706{,}3\ kJ/kg}$ (A-

And $\boxed{h_{in} = 3168{,}1\ kJ/kg}$ (Table A-6)

$$\implies m_{in} = 10 \times \frac{2706{,}3 - 1825{,}67}{3168{,}1 - 2706{,}3} = \boxed{19{,}07\ kg}$$

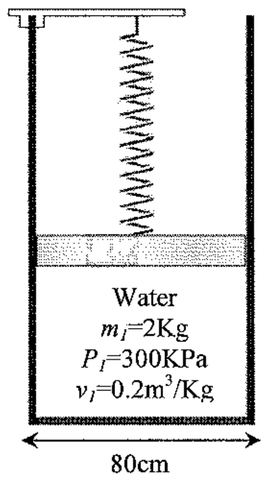
Problem III (30 points)

A frictionless piston-cylinder device initially contains m_1=2Kg of water at P_1=300KPa and v_1=0.2 m^3/Kg. At this state, a linear spring is touching the piston but exerting no force on it. The piston's diameter is 80cm, the atmospheric pressure 100KPa and the spring constant 100KN/m. Now, heat is added to the system until its volume doubles. Determine:

a) The force exerted by the spring on the piston at the final state.

b) The total work done by the water on the piston and the fraction of this work that is done against the spring to compress it.

c) The amount of heat that is added to the system.



Solution:

a) @ initial state: $V_1 = m_1 v_1 = 2 \times 0.2 = 0.4\,m^3$
@ final state: $V_2 = 2V_1 = 0.8\,m^3$
$\longrightarrow \Delta V = 0.4\,m^3$

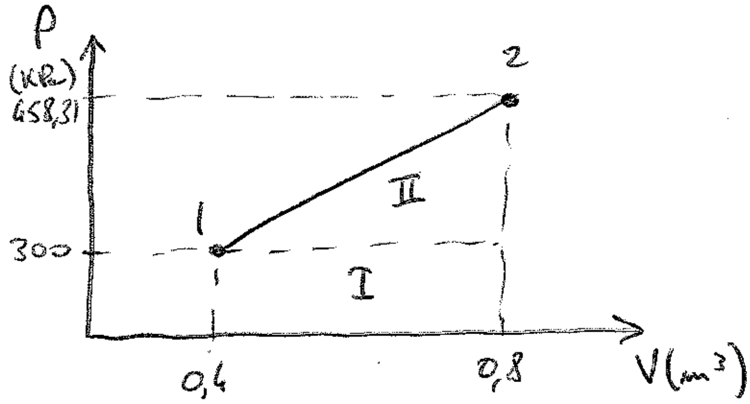
And the spring is compressed by $\Delta x = \frac{\Delta V}{A} = \frac{0.4}{\frac{\pi \times 0.8^2}{4}} = \boxed{0.79577\,m}$

$\Rightarrow F_{spring} = K \Delta x = 100 \times 0.79577 = \boxed{79.577\,KN}$

b) • Equilibrium of forces @ initial state:
$P_{atm} \times A + \text{Weight (Piston)} = P_1 A$

• Equilibrium of forces @ final state:
$P_2 A = F_{spring} + P_{atm} \times A + \text{Weight (Piston)} = F_{sp} + P_1 A$

$$\Rightarrow P_2 = \frac{F_{sp}}{A} + P_1 = \frac{79,577}{\frac{\pi \times 0,8^2}{4}} + 300 = \boxed{458,31\ KPa}$$



- The total work done by the water on the piston is:

$$W_b = \int_1^2 P\,dV = A(I) + A(II) = 300 \times 0,4 + \frac{158,31 \times 0,4}{2} = \boxed{151,662\ KJ}$$

- The fraction of this work done against the spring to compress it is nothing else but $A(II) = \frac{158,31 \times 0,4}{2} = \boxed{31,662\ KJ}$

c) 1st Law of thermodynamics:

$$E_{in} - E_{out} = \Delta E_{sys} \Rightarrow Q_{in} + \cancel{W_{in}} - \cancel{Q_{out}} - \cancel{W_{out}} - W_b = \Delta U + \cancel{\Delta KE} + \cancel{\Delta PE}$$

$$\rightarrow Q_{in} = \Delta U + W_b = m_1(u_2 - u_1) + W_b$$

@ 1: $P_1 = 300\ KPa$, $v_1 = 0,2\ m^3/Kg$ $\xrightarrow{A-5}$ $v_f < v_1 < v_g$ → sat. Lq. vap. mixture

$$\rightarrow x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{0,2 - 0,001073}{0,60582 - 0,001073} = 0,329$$

and $u_1 = u_f + x_1 u_{fg} = 561,11 + 0,329 \times 1982,1 = \boxed{1213,22\ KJ/Kg}$

@ 2: $P_2 = 458,31\ KPa$, $v_2 = 2v_1 = 0,4\ m^3/Kg$ $\xrightarrow{A-5}$ $v_g \simeq 0,4 = v_2 \Rightarrow u_2 = u_g = \boxed{2557,7\ KJ/Kg}$

P	v_g	u_g
450	0,41392	2557,1
458,31	≃ 0,4	2557,7
500	0,37483	2560,7

$$\Rightarrow Q_{in} = 2(2557,7 - 1213,22) + 151,662 = 2840,622\ KJ \approx \boxed{2,84\ MJ}$$